Pension Wealth, Preference Heterogeneity and the Impact of Retirement Policies

Søren Arnberg
Danish Ministry of Finance

Anne-Line Koch Helsø
Department of Economics, University of Copenhagen

Peter Stephensen
The Danish Institute for Economic Modelling and Forecasting, DREAM

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Abstract

We propose and estimate a structural retirement model with leisure preference heterogeneity using high-quality Danish register data with information about individual private pension wealth. We apply a non-parametric estimation technique to measure the heterogeneity in leisure preferences, and our estimates suggest that leisure preferences are widely distributed among the population. We use the model to study the extent to which increasing private retirement savings dampens the effect of retirement reforms targeted to increase labor supply. Our findings suggest that the effects of an increase in the normal retirement age by one year increases the average retirement age by 0.2 year if individuals hold zero or low wealth. The corresponding effect is more than halved once individuals hold savings equivalent to four or more years of pre-retirement earnings.

Keywords: Structural retirement model; Labor supply of seniors; Preference heterogeneity; Pension wealth; Policy evaluation

JEL Classifications: C51, C63, J14, J22, J26
1 Introduction

In virtually all developed countries, policy-makers seek ways to reform their pension system to ease the fiscal burden of increasing life expectancy. Many countries have done so by cutting pension benefits or postponing statutory retirement ages. At the same time, most OECD countries have introduced preferential tax treatments of pension savings to encourage people to save for retirement. As a result, private pension savings have increased substantially and will continue to do so (OECD. [2016]).

Increased private funding of pension payments reduce the need for government retirement transfers, and thereby help reduce the fiscal vulnerability of governments. However, larger private retirement savings also provide retirees with more flexibility to decide their own retirement age independent of the statutory retirement ages defined by the public retirement plans. As such, increased private retirement savings could also counteract the policies targeted to raise retirement ages, such as postponed statutory retirement ages or decreased pension benefits. This paper aims to explore the extent to which the increase in private pension wealth affects the effectiveness of such retirement policies.

To organize the empirical analyses, we propose a novel structural retirement model of senior worker’s retirement decision with heterogeneous leisure preferences, attrition, and improved health across generations\(^1\). The basic assumption underlying structural economic models is that people’s decisions are formed by their circumstances and preferences combined. In real life, however, we often observe that agents with identical circumstances behave differently, due to the self-evident fact that people are different and have different preferences. Allowing for preference heterogeneity, therefore, contributes with an important extra dimension to structural models, making them more flexible and realistic. One of the main contributions of this paper is that we propose and estimate a new retirement model where we allow for individuals to have different preferences for leisure when determining their optimal retirement age. We also show that the assumed preference heterogeneity significantly improves the model fit and alters the policy experiment conclusions compared to a model where all agents are assumed to have the same leisure preferences.

We estimate our model on full population Danish register data from 1996-2016, where we consider the retirement decisions of birth cohorts 1942-1954. Our data contain information about each person’s wealth in both retirement and non-retirement accounts, as well as highly detailed income information.

The Danish setting of our model has four main advantages: First, we have access to high-quality administrative information about the full population of Danes. Our data include detailed information about each senior’s pension and non-pension wealth, earnings, transfer

\(^1\)Our model is a development of the former model versions presented in Arnberg and Stephensen [2015] and Helso [2015]
income and transfer eligibility which are mainly 3rd-party reported. This unique data set is particularly suitable for studying the effect of financial incentives in retirement, where both earnings, non-pension wealth and private pension savings are extremely important factors. Compared to previous studies which are often based on the HRS (Health and Retirement Survey) for the U.S., our data contain a many more observations, which are also of higher quality since they are mainly 3rd-party reported. Second, there were several sharp pension reforms in Denmark, targeting higher retirement ages, which provide close to exogenous variation in retirement incentives across wealth groups. We use one of these reforms to externally validate our model fit. Third, the mandatory retirement savings in Denmark offset other savings to a very limited extent (e.g. Chetty et al. [2014] and Arnberg and Barslund [2014]), suggesting that pension wealth is almost exogenous. As some groups have faced mandatory savings schemes longer than others have, there is a significant dispersion in the level of private retirement savings of senior workers in Denmark. Since mandatory labor market pension saving schemes were expanded during the 1980s and 1990s, private pension savings have increased substantially, and will continue to do so during the next decades. In 2017, 85% of all workers contributed about 12-18% of their monthly wages to defined contribution private pension schemes making Denmark the OECD country with the largest ratio of assets to GDP in funded private pension arrangements (OECD. [2016]). Fourth, since private retirement plans in Denmark are predominantly DC (defined contribution) plans, we’re able to compute the future retirement income streams (as a function of the chosen retirement age) of almost everyone with very high accuracy.

Descriptive evidence suggests that financial incentives seem to be of high importance for the retirement decision of Danish seniors, who often retire immediately after meeting some eligibility criteria, and whose retirement patterns vary quite a lot with the level of private retirement wealth. We also observe that individuals respond to changes in eligibility criteria: when the statutory early retirement pension (ERP) age was increased from age 60 for cohort 1953 to 61 for cohort 1954 (2nd half), the corresponding drop in labor force participation at age 60 shifted almost entirely to age 61. As a result, the labor force participation at age 60 of cohort 1954 (2nd half) was 28 percentage points larger compared to cohort 1953. Furthermore, a means testing in ERP benefits with respect to private pension wealth can be avoided if retirement is postponed to at least two years after the ERP age. Our descriptive findings show that ERP eligible individuals with large retirement savings, who benefit the most from this rule, are also more likely to postpone their retirement age to 62.

Our proposed model shares many similarities with both the Option Value model and the Dynamic Programming model. The difference lies in our assumption that agents, at the age of 57, have perfect foresight about their future income, with the only uncertainty being the timing of their death. This also implies that we don’t account for any health or health cost related uncertainty. As such, our model mirrors the policy regime in Denmark and
many European countries, with free healthcare, guaranteed retirement benefits, and generous unemployment benefits. Our choice of model reflects a tradeoff between assumptions. The assumption that there is no future income or health uncertainty is of course a significant limitation of the model. But this simplifying assumption means that we can avoid the computational complexity of dealing with such uncertainties, and this enables us to instead relax the assumption that all agents share the same leisure preferences, which is the main contribution of our model. We estimate the model using a non-parametric estimation technique, which is a fixed-grid version of the Expectation Maximization (EM) algorithm as proposed by Train [2007]. We find substantial variation of the estimated leisure preferences within gender- and education specific groups. Our model fits the data well with reasonable parameter values. More importantly, our proposed model is able to fit the retirement response to a policy change in an external validation setting. We also find that our model performs much better compared to a similar model without preference heterogeneity, suggesting that the leisure preference heterogeneity assumption adds a significant contribution to the model. Previous studies have emphasized and studied the impact of heterogeneous leisure preferences on retirement behavior, see Gustman and Steinmeier [2005] and French and Jones [2011], but we are - to our knowledge - the first to propose and estimate a model with flexible and non-parametric heterogeneous leisure preferences.

Our work builds upon an extensive literature which studies the effects of financial incentives on senior workers’ labor supply decision. One strand of the literature models and estimates the retirement decision within a structural model framework and simulates the effects of various policy experiments. Examples are the Option Value model by Stock and Wise [1990] and Dynamic Programming models as devised by Rust and Phelan [1997]. In a dynamic life-cycle model, French [2005] finds that a 20% reduction in social security income raises the average retirement age of U.S. workers by three months. French and Jones [2011] find that an increase in the social security eligibility age in the US from 65 to 67 increases retirement ages by less than one month. Gustman and Steinmeier [2005] find that the effect of a two-year increase in the social security early eligibility age in the U.S. is two months. Another strand of the literature examines the ex-post evaluations of actual policy changes in reduced-form studies. Compared to the structural ex-ante simulation exercises, these studies often find larger effects of an increase in statutory retirement ages (Blundell et al. [2016]). Mastrobuoni [2009] finds that the mean retirement age of U.S. cohorts increased by one month when the normal retirement age was increased by two months. Another example is Lalive and Staubli [2014] who find that a one year increase in the normal retirement age of Swiss women delayed their labor market exit by as much as 7.9 months.

Mastrobuoni [2009] argues that many structural models underestimate the effects of an increase of the NRA because of measurement error (due to lack of precise data on retirement incentives) and because they do not account for social norms related to the NRA. Our model
includes a control for social norms related to the NRA, and we are convinced that our high-quality administrative data enables very precise calculations of the economic retirement incentives. Compared to previous structural models, our estimates are also in the higher end, however lower compared to most reduced form evidence. However, since the estimated magnitudes of policy effects very much depends on the country-specific retirement systems, direct comparisons are not possible.

We analyze the effects of different policy experiments and show how these vary for different levels of private retirement savings. First, we simulate a base-line experiment in which we abolish the early retirement program. As the early retirement program is drastically being phased out in Denmark, the baseline experiment reflects the retirement decision of future generations. We then contrast the retirement decisions in our baseline experiment to three additional experiments.

The first experiment is an increase in the normal retirement age by one year. We find that individuals with zero private retirement savings delay their retirement with 0.15-0.2 years once the statutory retirement age increases from 65 to 66. For individuals with private retirement wealth equivalent to four or more years of pre-retirement earnings, the same response is less than half the size.

The second experiment decreases the old age pension benefits by five percent, and for this experiment, we estimate a decline in the expected retirement age of roughly 0.08-0.09 years for those with no retirement savings and 0.07-0.06 years for those with large retirement savings. As such, the effect of a decrease in retirement benefits is much more stable across levels of retirement wealth as compared to an increase in the NRA.

The third experiment introduces a reduction in the old age pensions’ means testing with respect to private pension annuities when individuals retire one or more years after the statutory NRA. While low-wealth individuals are almost unaffected by this experiment, individuals holding large pension savings delay their retirement by up to 0.1 years.

In summary, our experiments show that the size of individual retirement savings can have important implications for the effect of different retirement reforms. For an increase in the NRA, we find a particularly large and negative effect of private retirement wealth on the reform’s ability to increase labor supply. It is possible, however, to reverse this effect - e.g. in a reform which mimics the two-year rule of the ERP scheme.

The paper is structured as follows: In Section 2, we describe the institutional settings defining the Danish retirement system. In Section 3, we describe the data and how we compute the income components which are not directly observed in the data, and in Section 4 we motivate our analysis with some descriptive figures of the data. In Section 5 we present our structural retirement model and Section 6 specifies how we estimate the model, including a detailed description of the fixed-grid version of the EM algorithm which we use to estimate the heterogeneous leisure preference parameters. Our estimation results are shown in Section
7, and in Section 8 we show how our model fit the data, together with an external validation test of the model. In Section 9, we run and evaluate three different policy experiments, and in Section 10, we compare our proposed model to a simpler version without leisure preference heterogeneity. Finally, Section 11 concludes.

2 Danish Institutional Settings

The following Section outlines the main components of the Danish retirement system, which mainly consists of three elements: the Early Retirement Pension, ERP (efterløn), the Old Age Pension, OAP (folkepension) and private pension savings which supplement the ERP/OAP benefits. The Section begins with a description of the policy rules which applied to birth cohorts 1942-1952 and ends with a description of how these rules are going to change for future retirees.

Early Retirement Pension (ERP)

The ERP is a voluntary scheme in which participants pay a quarterly membership fee (1.122 DKr in 2004) for at least 10 years to obtain eligibility. ERP benefits apply from age 60 until the normal retirement age of 65. ERP payouts resemble the level of unemployment benefits (about 200,000 DKr in 2018), and thereby it contains elements of both a funded and unfunded plan, where the government finances roughly 70% (Jørgensen [2014]), making it a quite attractive scheme. 69% of the cohort born in 1942 (92% of those in the labor force) were eligible for ERP benefits. Following a reform in 2011 which made the ERP program less lucrative, many workers have opted out of the ERP-scheme.

ERP benefits are means tested with respect to earnings, private pension payments and the accumulated amount of private pension savings. The two-year rule within the ERP program introduces a financial incentive for ERP members to postpone retirement further until age 62. If retirement is postponed by at least two years from age 60 to 62, the ERP benefit rate is increased from 91 to 100% of the UI benefit rate, and ERP benefits are no longer means tested with respect to accumulated private pension wealth. As such, the two-year incentive rule is especially relevant to those who hold large amounts of accumulated private retirement savings. The exact means testing in ERP benefits with respect to private retirement savings depends on their size, type (life annuity, rate- or capital pension) and category (employer or employee administrated) - see Section 2 for a further description. For a person with the most frequent type of savings (employer administrated life annuity), who have saved an amount corresponding to a lifelong annual payment of 50,000 DKr at retirement age 60, his ERP...
benefits at the 91% UI benefit rate are reduced by roughly 10% if retires before age 62, whether or not he decides to initiate his private pension payments. If he retires after age 62 and delays the initiation of his private pension payments until age 65, he will receive the full amount of the full ERP benefits at the 100% rate of UI benefits. The exact means testing rules of the ERP scheme with respect to private retirement savings are listed in Table 3. Had his accumulated life annuity savings corresponded to annual payments of 200,000 DKr (at retirement age 60), his ERP benefits would be reduced by roughly 50% if he retired before age 62 - a reduction which could be avoided if postponed his retirement age to or after age 62. When an ERP eligible individual fulfills the two-year rule but postpones his retirement even further, he receives one "portion" tax-free premium of 10,000 DKr (2004-level) for every four months of full-time work until the NRA at 65.

Old Age Pension (OAP)

The Danish OAP system is available to all Danish citizens aged 65 or above and is a fully government financed pay-as-you-go program available to all Danish citizens. It consists of a baseline amount (grundbeløb) and a supplement (pensionstillæg). The baseline annual amount (58,776 DKr in 2004) applies to all and is only means tested with respect to concurrent labor market income. The size of the OAP supplement varies by marital status (26,208 DKr in 2004 for married and 56,148 DKr for singles) and is also means tested by concurrent earnings. It is also further means tested by private pension payments to the recipient and the labor market status an earnings of his/her spouse. Unlike the ERP, the OAP is not means tested with respect to the accumulated amount of private pension savings. Seniors with poor financial circumstances can apply for additional benefits, e.g., housing allowances. For each year an individual postpones his/her retirement age beyond the NRA age at 65, his/her future OAP rate increases with 6% (with a maximum of 10 years).

Private Retirement Savings

Most jobs in Denmark are covered by collective agreements which often include an employer administrated contribution plan in which a fixed proportion of the monthly wages are paid into a defined contribution (DC) pension savings plan. These contribution plans are mandatory to all workers covered by the collective agreements, which counts about 85% of workers in 2018. The contribution rates are between 12-18 percent of the monthly wages (in 2018), where blue-collar workers typically contribute about 12 percent and white-collar workers between 15-18 percent. While the mandatory contribution plans started already in the 1950s for some high-skilled groups of workers, the contribution plans for large groups took form during the 1980s and 1990s, resulting in relatively large variation in private pension wealth for future cohorts of seniors. As such, private retirement savings have increased steadily
during the last 3-4 decades, and continue to do so until the 2050s-2060s when most retirees will have the saved for retirement during their entire careers. In addition to the mandatory employer administrated contribution plans, individuals can also voluntary save money for their retirement in employee administrated pension accounts, but these types of retirement savings only make up a small fraction of total private retirement savings.

The two main categories (employer and employee administrated savings accounts) each consists of three types of retirement savings: 1) life annuities (livrente), 2) term pension (ratepension) and 3) capital pension (kapitalpension). Life annuities guarantee a monthly payment from retirement until death, according to the accumulated amount. Term pensions are also paid out as annuities, but their duration only lasts between 10 and 25 years and can be initiated no later than age 77. Capital pensions are pension balances with no requirements on installment and are usually paid as a lump sum, no longer than 15 years after retirement.

As in most other OECD countries, the tax treatment of retirement savings provides a tax advantage when people save for retirement. While returns of regular non-retirement savings are taxed by approximately 33%, retirement savings are taxed by only 15.3%. However, private pension savings are means tested in public pension payments (ERP and OAP), which increases the effective taxation.

### Trends in statutory retirement ages

Figure 1 shows how the statutory retirement ages change across cohorts. Both the ERA and the NRA increase over time, and the length of the ERP Program decreases from five to three years, with a gradual abolition of the two-year rule. For individuals born later than 1960, the ERP scheme is three years long, and there is no two-year rule. Whereas the full ERP amount is increased to 100% of unemployment benefits, there is no reduction in the means testing of the ERP benefit with respect to private pension wealth, as were the case with the two-year rule for earlier cohorts. On the contrary, the means testing is further sharpened for these cohorts, from 60 to 80%, and the lower-limit allowance is removed. Due to larger reductions in ERP benefits by private retirement savings, the scheme is now much more unattractive for most workers, and the share of ERP members has dropped drastically. For the cohort born in 1970 or later, only 12% are members of the ERP-scheme, compared to 69% in the 1942 cohort.

### 3 Data

We base our analysis on administrative register data available from Statistics Denmark, covering the full population of Danes (approximately 5.5 million people) in 1996-2016 who were born between 1942-1954. Our data includes detailed yearly information about a wide
range of variables: earnings, pension savings, liquid wealth, transfer income, non-pension wealth, employment status, demographics, etc., which are mainly 3rd-party reported. For each person, income streams at age $a \in \{58, ..., 120\}$ is defined for each of the possible retirement ages, $r \in \{58, ..., 72\}$. While parts of the income streams can be directly observed in the data, we compute all future and counter-factual income streams based on each person’s observed benefit entitlements, retirement savings and earnings history. Given the richness of our data, we are able to compute the income streams with a high level of detail and accuracy. In the following, we describe how we observe/compute each of the financial components, how we measure the actual retirement ages, and how we define our sample:

**Income**

*Earnings*: We observe annual labor earnings directly in data for both wage earners and self-employed up until retirement, and we assume that the counterfactual earnings equal the average of the latest three observed earnings prior to retirement, with a wage inflation of 3.2%. We also include unemployment insurance benefits in our earnings definition. Our binary retirement assumption results in individuals having zero earnings in retirement. As our sample only includes individuals with full-time employment at age 54, we assume that all individuals receive 0 transfers, excluding unemployment insurance benefits, before retirement.

*Early Retirement Pension (ERP)*: We observe if an individual is entitled to receive early retirement benefits or not at age 59, and we compute the ERP benefit rates according to the
rules described in Section 2. We assume that all ERP eligible individuals are full-time insured, that they are eligible from age 60, and thereby meet the two-year-rule requirement at age 62. Appendix Table 3 outlines in detail how the ERP benefit is means tested with respect to different types of retirement savings. Individuals who retire after age 62, and who are thereby fulfilling the two-year rule, are assumed to postpone any individual pension payments until age 65 in order to avoid any reductions in the ERP benefits. If a person fulfills the two-year rule but delays retirement, he will receive a tax-free bonus of approximately 40,000 DKr for each year of postponement.

*Old Age Pension (OAP)* We model the OAP benefit payments according to the rules described in Section 2. We assume that all individuals older than 65 are entitled to receive the OAP. We assume that individuals only choose to receive OAP after their retirement. As we assume 0 earnings in retirement, all individuals receive the full OAP baseline amount. The amount of supplemental OAP depends on the cohabiting partner’s labor market status and earnings and is reduced with the recipient’s level of total income (including private pension payouts). We consider the latest observed labor market status and earnings of each person’s partner and assume that individuals do not change their partner after 2018. If the partner’s retirement age is unobserved in data, we assume it to be 65. We further assume that the partner does not die. While these assumptions might seem coarse, recall that the partner status only influences our model through the supplemental OAP benefit rate. As for the remaining transfer income types, future benefit rates are assumed to grow with an assumed inflation rate of 3.2%.

The exact retirement age of the spouse is not necessarily observed. In that case, it is set to the default retirement age, 65. The latest observed salary of the spouse is extrapolated with wage inflation \( \pi_{\text{wage}} = 3.2\% \) until the actual or assumed retirement. Some individuals lose their spouse and/or get a new spouse during the age interval of 60 to 67. You can argue both against and in favor of including observed changes in partner status when computing the future income streams. Whether individuals are able to predict divorces, the death of a partner, meeting a new partner, etc. is a delicate matter. We include all observed changes in partner status in accordance with the perfect foresight assumption.

*Other Retirement Benefits* that we also compute and include in our income definition include housing benefits (“boligsikring”) and older check (“ældrecheck”). All future transfer income is assumed to grow with an assumed inflation rate of 3.2%.

*Taxes:* We apply the actual tax rules which applied up until, and including 2018. For later years, we apply the 2018 rules where we let the limit amounts for the different progressive tax-levels increase with the wage inflation of 3.2%. As such, we implicitly assume that individuals also have perfect foresight with respect to the different tax reforms which took place between 1996 and 2018.
Retirement Wealth and Retirement Income

We observe the accumulated amounts and types of pension wealth for all individuals when they are 59.5 years old. These are 3rd party reported by the financial institutions, and are therefore highly reliable. We also observe what types of pension savings each person holds, which enables us to compute the payment schemes and corresponding benefit deductions with high accuracy. Some pension savings are observed as the deposited amount and others as the annual commitment given retirement at age 60. We assume that individuals save for their retirement as long as they work. If the actual contribution rate for an individual is unobserved, we assume a contribution rate equal to the average of observed contribution rates from when the individual was 54 years old, until his/her observed retirement age.

Capital Pension: We assume that Capital Pensions (CP) are paid as lump sums during the first year of retirement. We assume that the capital pension deposit grows with the annual interest rate $i_d = 4.75\%$ in the period prior to retirement. All interest gains on retirement savings are taxed with the so-called PAL tax, $\tau_{PAL} = 15.3\%$. $CP_{59}$ is observed directly in the data, and the subsequent years are computed as

$$CP_a = CP_{a-1} \times (1 + i_d(1 - \tau_{PAL})) + \Delta CP_{a-1} \text{ for } a \leq r$$

where $\Delta CP_{a-1}$ denotes the contributed amount to the capital pension at age $a - 1$.

Term Pension: We assume that all terms pension payments are equally distributed through annuities of 10 years, such that the payment size equals 10% of the deposited amount at the retirement age, growing with interests $1 + i_r(1 - \tau_{PAL})$ each year. Payments start at the year of retirement with exemption of ERP eligible individuals who retire at age 63 or 64. They are assumed to postpone the payments until age 65 to avoid reductions in ERP benefits.

Life annuities: We observe life annuities both as total deposited value ($LA_{TOT}$) and as annual commitments given retirement at age 60 ($LA_{PAY}$). Assuming that the deposited values at all times should equal the present value of future annuity payments, we get the following correspondence between the two figures:

$$LA_{PAY}^a = \frac{LA_{TOT}^a}{\sum_{i=a}^{100} (1 - \mu_i) \times \left(\frac{1 + i_d(1 - \tau_{PAL})}{1 + \pi_{wage}}\right)^{-i-(a+1)}}$$

The total committed amount, $LA_{TOT}^a$, is assumed to follow same development as the capital pension such that

$$LA_{TOT}^a = LA_{TOT}^{a-1} \times (1 + i_d(1 - \tau_{PAL})) + \Delta LA_{TOT}^{a-1} \text{ for } a \leq r$$

where $\Delta LA_{TOT}^{a-1}$ denotes the amount contributed to the life annuity savings at age $a -$
1. $L_{a}^{TOT}$ denotes the age $a$ fixed-price value of the total commitment and $\mu_{i}$ the death probability at age $i$. The interests gained on the deposited value are assumed to equal the interest rate on deposits $i_{d} = 4.75\%$. We let $\pi_{wage}$ denote the wage inflation, set to growth ($g=1.5\%$) times price inflation $\pi_{price} = 1.75\%$, such that $\pi_{wage} = 1.015 \times 1.0175 = 1.032 = 3.2\%$.

**Retirement definition**

To identify the chosen retirement ages of our sample, we also consider the DREAM-register which contains weekly information about all public transfer payments made to each person, including ERP and OAP benefits. We combine these weekly observations with annual earnings and income observations to measure people’s chosen retirement ages. We define an individual to be retired if at least one of the below statements are true: 1) Receives ERP benefits, 2) When the yearly salary is less than half of the pre-retirement salary for two years in a row. We define the pre-retirement salary as the average annual salary observed at ages 55-57. We prefer this relative income threshold rather than an absolute threshold to account for differences in worker productivity. Retirement is assumed to be an absorbing and binary state, and as such, it suffers from two limitations. First, some people may return to employment after retirement. Second, some people may still have little employment even though their yearly wages “permanently” become less than half of their initial wage.

**Sample Definition**

Covering the period from 1996 to 2016, we observe individuals born in 1942 to 1954 from when they are 55 years old. As such, we observe cohort 1942 until they are 74 years old, while we only observe cohort 1952 until they are 64 years old. The estimation sample includes cohorts 1942-1952, whereas the remaining cohorts are used for out-of-sample validation and forecasts. We restrict the analysis to cover all individuals who at age 57 face an actual retirement decision (i.e. not retired people), which amounts to all individuals who are observed with earnings above 90.000 DKr (2001-amount), excluding individuals on disability- and transition benefits and civil servants. Table 1 outlines how the data size decreases when the different groups are discarded from the sample. We end up with roughly 53\% of the population for cohort 1942, and the coverage increases over the cohorts such that our sample includes 69\% of the population born in 1953 - mostly due to increased labor force participation at age 57 (increasing from 63 to 74\%). Individuals who are not present in data throughout all 18 consecutive years (age 58 to 72) are not discarded from the analysis. Appendix D describes how data censoring is handled throughout the estimation process.

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3Civil servants refer to those eligible for the Danish equivalence of a defined benefit plan in the US ("tjenestemandspension").
Table 1: Sample Selection

<table>
<thead>
<tr>
<th>Cohort</th>
<th>Initial sample Size</th>
<th>Excl. not in labor force</th>
<th>Excl. disab. pens.</th>
<th>Excl. civil serv.</th>
<th>Share of initial sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1942</td>
<td>65998</td>
<td>41609</td>
<td>40294</td>
<td>33675</td>
<td>0.53</td>
</tr>
<tr>
<td>1943</td>
<td>69767</td>
<td>45126</td>
<td>43663</td>
<td>36217</td>
<td>0.56</td>
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<tr>
<td>1944</td>
<td>75475</td>
<td>50155</td>
<td>48430</td>
<td>40320</td>
<td>0.58</td>
</tr>
<tr>
<td>1945</td>
<td>78956</td>
<td>53921</td>
<td>51970</td>
<td>43050</td>
<td>0.60</td>
</tr>
<tr>
<td>1946</td>
<td>81348</td>
<td>56962</td>
<td>54906</td>
<td>45408</td>
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<tr>
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<td>54975</td>
<td>45958</td>
<td>0.64</td>
</tr>
<tr>
<td>1948</td>
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<td>54041</td>
<td>52072</td>
<td>43107</td>
<td>0.65</td>
</tr>
<tr>
<td>1949</td>
<td>69714</td>
<td>50872</td>
<td>48863</td>
<td>40424</td>
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<tr>
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<td>51836</td>
<td>50100</td>
<td>42651</td>
<td>0.69</td>
</tr>
</tbody>
</table>

The table shows how the size of our main sample varies after we exclude individuals who are not facing a “standard” retirement decision, by cohort. The initial sample includes the full population of Danes who were alive and living in Denmark at age 57. The Column “Excl. not in labor force” shows the sample size once everyone who is not working (defined by annual earnings larger than 90,000 DKr (2001-value)) are excluded. The Column “Excl. disab. pens.” shows the remaining sample size once also those who retire on disability insurance after age 57 are excluded. The Column “Excl. civil serv.” excludes those individuals who ever paid into a “civil servant” retirement account. We exclude these individuals as we’re not able to properly observe their private retirement entitlements (today, the civil servant arrangement is much smaller in scale).

4 Descriptive Statistics

4.1 Trends in retirement

Figure 2 shows the cohort-specific labor force participation rates of our main sample as specified in Table 1. The figure shows the cohort-specific developments in labor force participation from age 57 to 71. For cohorts 1942-1953, we see that employment shares drop at the ERP age (60), at age 62 where the two-year-rule is activated and again - however smaller in size - at the normal retirement age 65. For the cohort born in 1954 (2nd half), we see that the drop in employment share is shifted to the new ERP eligibility age 61. At age 60, the labor force participation of cohort 1954 (2nd half) was 28 percentage points larger compared to cohort 1953.
Figure 2: Labor Force Participation by Cohort, Estimation Sample

The figure plots the labor force participation rate at ages 57-71 for different cohorts for the subgroup of the population who were in the labor force at age 57. (2) indicates that the plot only covers those born in the second half of the year (different statutory ages apply to individuals born in the first half of these cohorts). Based on own calculations on full-population Danish register data, using the retirement age definition described in Section 3.

Figure 2 shows an upward parallel shift in employment shares across cohorts, meaning that the younger cohorts tend to retire later, even though the statutory retirement ages are unchanged for the cohorts 1942-1953.

Figure 3 depicts the retirement ages of individuals who retire on the early retirement scheme with a monthly precision for the birth cohorts 1942-45. Most individuals retire immediately after their birthday from which they become eligible for either ERP (age 60) or from which they can benefit from the two-year rule (62). The fact that individuals tend to retire at these favorable ages, and immediately after their birthdays, suggests that financial incentives are essential when they time their retirement age. It could also suggest that social norms and/or reference point effects are at stake.

Figure 4 illustrates how the retirement wealth of seniors workers in Denmark increases over time. The figure shows the observed/predicted distribution of accumulated retirement savings for different cohorts, measured 7 years before the NRA. It clearly shows that the level of retirement savings are increasing over time, reflecting the maturation of the mandatory contribution plans, which were introduced in the 1990s for most workers. As such, younger cohorts have accumulated their pension wealth during more years. An increase in expected NRAs also explain some of the increase in retirement savings.
The histogram shows the distribution of the precise retirement age of the seniors born in 1942-1945, who retire on Early Retirement Pension. Data are from the DREAM database (100% of the population) where we can identify the exact week in which an individual starts to collect ERP benefits.

This figure shows the distribution of accumulated retirement savings of seniors who are working 7 years prior to their respective NRA, as measured by annual income for different cohorts. Retirement savings are measured six years prior to the NRA, divided by average annual income 7-10 years prior to the NRA. The distributions for cohort 1942, 1947 and 1953 are calculated using administrative register data, while the distributions for cohorts 1979 and 1997 are calculated using simulated data from the Danish Ministry of Finance’s micro simulation model MFI. MFI models the incomes and pensions savings of future generations, assuming that individuals will continue to contribute to mandatory pension saving schemes with current contribution rates. Thus, the model captures the maturation of private pension wealth.
4.2 Lessons from the ERP-scheme

Figure 5 illustrates the exit rate distribution across ages for individuals with different levels of retirement wealth. The right figure shows the exit rates of people who are not entitled to ERP-benefits. As expected we see that those with large pension wealth are more likely to self-finance an earlier retirement prior to the NRA. Approximately 40% of those who are not members of the ERP program decide to self-finance an earlier retirement age, and this share increases with the level of retirement wealth. Different means testing and eligibility rules apply to the ERP-scheme and the old age pension as described in Section 2. Where the early retirement scheme induces individuals with large private pension savings to postpone their retirement until age 62. The exit rates in Figure 5 show that individuals respond to these incentives. In the group of individuals entitled to early retirement benefits, those with high private pension wealth retire later than those with smaller private pension savings. Thus, the two-year rule of the ERP-scheme reverses the negative retirement effect of private pension wealth.

Figure 5: Observed exit rates for different levels of pension wealth, ERP and Non-ERP eligible

![Graph showing exit rates for different pension wealth categories.](image)

The two figures plot the observed exit rates (share who exit the labor force) at ages 58-72 for our main sample (see Table 1), cohorts 1942-1944. The quartiles of the retirement savings distribution are computed within cohorts.

Of course, earnings levels also explain some of the heterogeneity in retirement behavior. To check that the findings in Figure 5 also apply to individuals with similar income levels, we show similar plots which are split by earnings quartile. Appendix Figure 17 confirms that high pension wealth individuals who are ERP-eligible tend to retire later than the low pension wealth ERP eligible individuals - conditional on them being in the same income quartile. For those who are not eligible for ERP benefits, we also observe that the high-wealth individuals tend to retire earlier than low-wealth individuals, conditional on them being in the same earnings quartile, see Appendix Figure 18. The described patterns hold
across different gender- and education groups.
5 The Model

We model retirement as an absorbing state with the binary option of either working full-time or being retired at a given age. The model is derived in a random utility model framework with $N$ utility-maximizing agents who at age $a = 57$ decides when to retire, $r \in \{58, ..., 72\}$. Evaluated at age 57, the utility of retiring at age $r$ consists of a deterministic and a stochastic component:

$$U(r) = V(r) + \sigma \epsilon_r$$

(1)

$V(r)$ denotes the present value of discounted lifetime utility of consumption obtained from retirement age $r$. The retirement age dependent unobserved heterogeneity $\epsilon_r$ is assumed i.i.d. extreme value distributed with scaling parameter $\sigma$. $\epsilon_r$ is known to the agent, but not to the econometrician, and reflects individual preferences not captured by the model, such as health, behavior of the spouse, etc. The scale parameter $\sigma$ determines the weight put on the deterministic relative to the stochastic part of the utility function. If $\sigma = 0$, economic incentives alone rule the retirement decision. Large values of $\sigma$, on the other hand, implies that the retirement decision, from the econometrician’s perspective, is random.

We assume that agents have perfect foresight with respect to their future financial situation - they know with certainty their future salaries, interest rates, pension payments, benefit entitlements, etc. The only uncertainty is with respect to death, where age- and gender-specific death probabilities are known to the individuals. As such, we do not account for any income, health or health-cost related uncertainty. Such assumptions are less critical in a Danish setting with free universal health care and generous unemployment benefits, and these assumptions collapse the retirement decision problem into a discrete choice problem between the different retirement ages $r \in \{58, ..., 72\}$.

We assume CRRA utility with relative risk parameter $\rho$. $V(r)$ is given by

$$V(r) = \sum_{a=58}^{120} \frac{(\gamma(r, a|k, \alpha)c_a)^{1-\rho}}{1-\rho} D_a$$

(2)

where $D_a$ is the mortality-adjusted discount factor:

$$D_a = \beta^{a-57} \prod_{s=58}^{a} (1 - \mu_s).$$

Here, $\mu_s$ denotes the death probability at age $s$, and $\beta$ denotes the usual discount factor. $c_a$ denotes the agent’s chosen level of consumption in period $a$, given the optimal consumption problem described below. Consumption $c_a$ is scaled with the age- and retirement-age dependent function $\gamma(r, a|k, \alpha)$, whose shape is defined by the leisure preference parameter $k$, and $\alpha$ which we will refer to as the “attrition parameter”.
The functional form of $\gamma(r, a|k, \alpha)$ is given by

$$
\gamma(r, a|k, \alpha) = \begin{cases} 
e^{-\alpha a^2} & \text{for } a < r \\
 k \cdot e^{-\alpha r^2} & \text{for } a \geq r
\end{cases}
$$

$\gamma$ scales the utility of consumption, and figure 6 illustrates how its shape depends on the parameters $\alpha$ and $k$.

The leisure preference parameter $k$ measures how an agent values consumption in retirement relative to while working, and as such we think of $k$ as a measure of the agent’s preference for leisure (or disutility from working). The $k$ parameter is similar to that in the option value proposed by Stock and Wise [1990]. If $k$ is 1.1, the consumer values one unit of consumption 1.1 times higher once retired. A key feature of our model is that we allow $k$ to vary for each person, and estimate its population distribution rather than a simple point estimate. We think of leisure preferences quite broadly, as a concept that also captures other residual individual preferences related to retirement. That could be the value of joint retirement with a spouse, the part of an individual’s health status that is not declining with more years in the labor force, etc.

Another central preference parameter is $\alpha$, which permanently decreases an individual’s utility level after retirement - and more, the longer the retirement is postponed, see Figure 6. We think of $\alpha$ as capturing some kind of “attrition” related to working, and is therefore closely related to the agent’s health status. Once retired, the utility level remains constant at the decreased value throughout the remaining of the agent’s life. We allow for the attrition parameter to depend linearly on the cohort $b$ in order to capture a trend in the health status across generations:

$$
\alpha = \alpha_0 + b\alpha_1
$$

Agents with wealth endowment $W_a$, income $y_a$, and consumption $c_a$ at age $a$ face the budget

Figure 6: The parameter $\gamma_a(r)$

(a) Constant utility

(b) Variable utility

(c) Change of cohort
restriction

\[ W_a = (1 + (1 - \tau)i_a)W_{a-1} + y_a(r) - c_a, \]  

where \( \tau \) denotes a tax on capital income and \( i_a \) denotes the age-dependent and actuarial fair interest rate, dependent on the risk-free interest rate \( i \) and the death probability at age \( a \), \( \mu_a \):

\[ 1 + i_a = \frac{1 + i}{1 - \mu_a} \]  

The future income stream \( y_a(r) \), is assumed to be known by the individual with certainty for each possible \( r \). It is determined by the observed earnings level of the individual, his/her accumulated retirement savings, benefit entitlements etc., as described in Section 2.

Under these assumptions, standard calculus yields a closed-form solution to the optimal consumption problem, which we show in Appendix B. An agent who maximizes the utility function stated in 1 for a given retirement age \( r \), and given the budget restriction in equation 3 and interest rate in equation 4, will have the indirect utility function:

\[ V(r) = \frac{(W_{57} + H(r))^{1-\rho}}{1 - \rho} \]  

where \( W_{57} \) is wealth at the beginning of age 58. \( H(r) \) and \( P(r) \) are given by

\[ H(r) = \sum_{a=58}^{120} y_a(r) R_a \]

\[ P(r) = \left[ \sum_{a=58}^{120} \gamma_a(r) \frac{1}{\rho} \frac{1}{D_a \left[ \frac{\rho - 1}{\rho} \right]} \right]^{\frac{\rho}{\rho - 1}} \]

where \( R_a \) is defined as

\[ R_a = \prod_{s=58}^{a} \frac{1}{1 + (1 - \tau)i_a} \]

As such, \( H(r) \) measures the future discounted income at the beginning of age 58 for chosen retirement age \( r \), and \( P(r) \) is the corresponding CES price index of future discounting factors. In a similar model, Helsø [2015] also models the consumption-savings decision separately, relaxing the assumption of a symmetrical interest rate function. Whereas this addition complicates the computation of the indirect utility function, it did not contribute significantly with respect to the model’s performance.

As per our assumption that \( \epsilon_r \) is iid. extreme value distributed, the probability of retirement at age \( r \) is computed by the logit specification

\[ Prob(r) = \frac{exp(V(r))}{\sum_{r'=55}^{72} exp(V(r'))} \]
We observe a significant number of individuals who choose to retire at the statutory old age pension retirement age, despite relatively weak financial incentives at this specific retirement age. A common explanation is that the NRA provides a focal/reference point for decision-making and shapes the social norms about what is the appropriate retirement age. This effect has been studied in e.g. Behaghel and Blau [2012] and Seibold [2017]. To include such a focal point effect, we estimate a slightly different version of the model, in which the indirect utility is given by

\[ V(r) = V(r) + d_{65}1_{r=65} \] (7)

where \( d_{65} \) measures a jump in utility if the chosen retirement age is exactly equal to the statutory old age pension age, which is 65 years in the estimation sample.\(^4\)

Consequently, we slightly alter the utility equation described in equation 1 such that

\[ U(r) = V(r) + \sigma \epsilon_r = V(r) + d_{65}1_{r=65} + \sigma \epsilon_r \] (8)

The probability of retirement basically depend on three types of variables: 1) preference parameters that are assumed to be identical for all consumers \( \Theta = (\alpha_0, \alpha_1, \beta, \sigma, \rho, d_{65}^0, d_{65}^1) \), 2) the heterogeneous leisure preference parameter \( k \), and 3) variables that characterize the financial situation of each individual, \( x^j = (V(58), ..., V(72)) \). The indirect utility function given by (5) and (7) is therefore a function of these variables, and we can write the retirement probability as

\[ \text{Prob}(r|\Theta, x, k) = \frac{\exp \left( V(r|\Theta, x, k) \right)}{\sum_{r'=58}^{72} \exp \left( V(r'|\Theta, x, k) \right)} \] (9)

6 Estimation

For each agent \( j \in 1, ... n \), we observe \((r^j, x^j)\), where \( r^j \) is person \( j \)'s actual retirement age, and \( x_j = (W^j_{57}, H(58)^j, ..., H(72)^j) \) comprise his/her financial data (see Section 5 equation 5). We estimate the homogeneous model parameters \( \Theta = (\alpha, \beta, \sigma, \rho, d_{65}) \) together with the population distribution of the heterogeneous preference parameter, \( p(k) \), in a nested 2-step procedure: first we assume some value of the homogeneous model parameters, \( \hat{\Theta} \). Given \( \hat{\Theta} \) and the observed data, \((r^j, x^j)\), we’re then able to compute the corresponding population distribution of the leisure preference parameter \( p(k|\hat{\Theta}) \). We do this using a fixed-grid version of the Expectation Maximization (EM) algorithm as described in the following. Given the model parameters \( (\hat{\Theta}, p(k|\hat{\Theta})) \), we can compute the likelihood function, which is the joint

\(^4\)We allow the value of \( d_{65} \) to vary between individuals born before and after 1946 to account for a temporary tax-reduction policy ("skattenedslaget") which made it more attractive for individuals born in 1946-1952 to retire on or after the statutory retirement age.
probability of all observed retirement ages:

$$L \left( \hat{\Theta}, p(k|\hat{\Theta}) | x, r \right) = \prod_{j=1}^{n} \text{Prob} (r^j | \hat{\Theta}, x^j, p(k|\hat{\Theta}))$$

Then we consider a new set of model parameters $\hat{\Theta}$, and repeat the 2-step procedure. We continue to do so, until we have obtained the maximum likelihood. We use the non-linear optimization routine NLM in R to search through the parameter space of $\Theta$.

We compute $p(k)$ using the fixed-grid version of the Expectation Maximization (EM) algorithm as proposed in Train [2008] Section 6. The EM algorithm was originally a procedure developed for dealing with missing data, proposed by Dempster et al. [1977]. With this method, we estimate the population distribution of the preference parameter $p(k)$ as a discrete distribution, where we, for a fixed grid of k-values, $k_{\text{min}}, ..., k_{\text{max}}$, estimate the probability mass (or share) in each grid point, $p(k_{\text{min}}), ..., p(k_{\text{max}})$. As an alternative to the fixed-grid version of the EM-algorithm, one could also estimate $p(k)$ numerically, using the method proposed by Fox et al. [2016]

Assume that we know the homogeneous model parameters $\Theta$. The probability that individual $j$ retires at age $r^j$ with financial circumstances $x^j$, conditional on a given $k' \in \{k_{\text{min}}, ..., k_{\text{max}}\}$, is given by the logit specification

$$\text{Prob} \left( r^j | \Theta, x^j, k' \right) = \frac{\exp \left( V(r^j | \Theta, x^j, k') \right)}{\sum_{r'=58}^{72} \exp \left( V(r' | \Theta, x^j, k') \right)}$$  \hspace{1cm} (10)

where $\bar{V}(r)$ denotes the indirect utility of retiring at age $r$ given in equation 7. Applying Bayes’ rule to $r^j$ and $k$ in the above equation, we’re able to deduct the probability that individual $j$ has preference parameter $k$

$$\text{Prob}(k'|\Theta, x^j, r^j) = \frac{\text{Prob} \left( r^j | \Theta, x^j, k' \right) \cdot p(k'|\Theta)}{\text{Prob}(r^j | \Theta, x^j)}$$  \hspace{1cm} (11)

We compute the denominator in equation 10 by marginalizing out the conditional variable $k$ in equation 11 using numerical integration:

$$\text{Prob}(r^j | \Theta, x^j) = \sum_{k''=k_{\text{min}}}^{k_{\text{max}}} \text{Prob} \left( r^j | \Theta, x^j, k'' \right) \cdot p(k''|\Theta)$$

However, as we do not know $p(k')$, we cannot directly compute 10. Instead, we rely on the EM-algorithm, which enables us to derive $\text{Prob}(k'|r^j, x^j, \Theta)$ by an iterative recursion. Let $i$ denote the iteration. We start with some initial guess of the probability mass of $k'$: $p(k')^0$. Then we repeatedly update its value by the formula:
\[ p(k' | \Theta)^{i+1} = \frac{1}{n} \sum_{j=1}^{n} \frac{\text{Prob}(r^j | \Theta, x^j, k') p(k' | \Theta)^i}{\text{Prob}(r^j | \Theta, x^j)} \]  

(12)

The EM recursion is repeated until convergence. In practice, we assume an even-spaced \( k \)-grid, where we use cross-validation to find the optimal distance between the gridpoints. Standard errors are bootstrapped as suggested by Train [2008]. The nonparametric estimation of the \( k \)-distribution relies on the assumption that \( k \) and \( x \) are independent. Recall that \( x^j = (V(58), ..., V(72)) \) is a function of initial wealth and income streams for all retirement ages. It seems likely that this assumption does not hold, since individuals with high leisure preferences are also likely to have lower income and/or wealth. Helsø [2015], however, examines how a violation of the assumption affect estimation within a data simulation, and finds that an even extremely high degrees of correlation (±0.5) only have limited implications for the corresponding estimation bias of the \( k \)-distribution. In addition, we estimate the model separately by gender and education groups, which mitigates this concern as we achieve smaller variation in \( x^j \) within each group. The correlation between an individual’s estimated expected \( k \) and \( V(58) \) varies between 0.06 and 0.177 in the different gender- and education groups.

A step-by-step description of the estimation algorithm is included in the following text box.
Estimation Algorithm

1. Observe the actual retirement ages \( r^j \) and financial situations \( x^j \) of individuals \( j = 1, \ldots, n \).

2. Set the fixed grid points of \( k \) within some boundary values, \( k_{min} \) and \( k_{max} \) and assume some initial distribution \( p(k)^0 \) on the defined interval (e.g. a flat uniform distribution).

3. Consider some set of homogeneous model parameters \( \hat{\Theta} = (\hat{\alpha}, \hat{\beta}, \hat{\sigma}, \hat{\rho}, \hat{d}_{65}) \).

3(a) For each person and each \( k' \in \{k_{min}, \ldots, k_{max}\} \), compute \( \text{Prob}(r^j|\Theta, x^j, k') \).

3(b) For each person and each \( k' \in \{k_{min}, \ldots, k_{max}\} \), compute \( \text{Prob}(k'|\Theta, x^j, r^j)^i = \frac{\text{Prob}(r^j|\Theta, x^j, k')p(k'|\Theta)^{i-1}}{\sum_{k''=k_{min}}^{k_{max}} \text{Prob}(r^j|\Theta, x^j, k'')p(k''|\Theta)^{i-1}} \).

3(c) For \( k' \in \{k_{min}, \ldots, k_{max}\} \), update the population distribution such that \( p(k'|\hat{\Theta})^i = \frac{1}{n} \sum_{j=1}^{n} \text{Prob}(k'|r^j, x^j, \Theta)^i \) for \( k' \in \{k_{min}, \ldots, k_{max}\} \).

3(d) Repeat step 3(b)-3(c) until convergence of \( p(k|\hat{\Theta}) \).

4. Compute log likelihood \( L(\Theta, p(k|\hat{\Theta})|x, \Theta) \).

5. Return to step 3 and consider a new guess of parameters \( \hat{\Theta} \), until likelihood function is maximized.

7 Estimation Results

We use a random 33% sample of the full-population sample described in Table 1 to estimate the model separately for eight gender- and education specific groups, covering the cohorts born in 1942 to 1952. Since our data end in 2016, we observe cohort 1942 until they are 73 year old, while we only observe cohort 1952 until age 62. Other individuals might also die before we observe their chosen retirement age. For the individuals where we do not observe their retirement, we will use the information that they did not retire at the observed retirement ages. Appendix Section D elaborates on how we deal with this censoring. The homogeneous parameters \( \Theta = (\alpha, \beta, \sigma, \rho, d_{65}) \) are estimated in an outer log likelihood optimization search routine, while for each guess of \( \hat{\Theta} \), the corresponding distribution of \( k \) is estimated in a nested iterative fixed point algorithm described in Section 6. We let \( k_{min} = 0.05 \) and \( k_{max} = 3.05 \) with a step size of 0.1. Table 2 presents the maximum-likelihood estimated parameters \( \Theta = (\alpha, \beta, \sigma, \rho, d_{65}) \).
The estimated values of the attrition parameter $\alpha_0$ are largest for males, and their magnitudes indicate that attrition is playing a key role in our specified model. For men born in cohort 1942 with vocational education, consumption will be down-scaled by a factor of 0.75 if retirement is postponed to age 62 and 0.47 if retirement is postponed to age 65, and this decline in utility persists throughout life. For women with vocational training, consumption is down-scaled by a factor of 0.84 if retirement is postponed to age 62 and 0.63 when delayed to age 65. The negative values of $\alpha_1$ which specifies a cohort trend in attrition, is largest for men, suggest that attrition levels are decreasing for younger cohorts and that the gender gap in attrition is shrinking over time. The estimated discount factors, $\beta$, lie within a reasonable range from 0.937-0.973. Relative risk aversion coefficients, $\rho$, are close to 1 for all groups, and slightly larger for women, indicating that individuals are moderately risk averse. This is in line with the findings in Chetty [2006]. It is in line with what has been found.
in similar studies, e.g. Rust and Phelan [1997] (1.07) and Blau and Gilleskie [2003] (0.95), but is smaller than other studies, e.g. French [2005] (2.2-5.1) and French and Jones [2011] (7.49). $\sigma$ measures the scale of the extreme value type I error terms, and thereby weights the deterministic and stochastic components of our model as specified in equation 8. As such, a small value of $\sigma$ indicates that the financial incentives are important for the retirement decision. However, the size of $\sigma$ also reflects the overall variation of the deterministic part of the utility $V(r)$, as specified in equation 8, which do not only depend on the remaining homogeneous model parameters, but also varies with $k$, and as such its interpretation is not straightforward.

As expected, the social norm dummy for cohorts 1942-46 ($d_{65}^{0}$) is larger than the social norm for the cohorts 1947-1952 ($d_{65}^{1}$). This is due to the temporary tax-reform “skattened-slaget”, which made it more attractive for those born in 1947-1952 to retire on or later than the NRA. While the incentive did not lead to later retirement ages for these cohorts (few people understood this incentive), the financial incentives alone can explain a larger share of the individuals retiring at the NRA. The size of these social norm dummies should be interpreted in relation to the estimated scale parameter of the error term in the utility function, specified in equation 8, $\sigma$. The estimated parameter values of $d_{65}^{0}$ are roughly 1-2 times larger than $\sigma$, implying that the utility jump which an agent receives if he/she retires at the NRA (which we interpret as the magnitude of the social norms) is slightly larger than the overall variation of the agent’s stochastic utility component, $\epsilon$.

The homogeneous parameter estimates should be interpreted in the context of the estimated distributions of the heterogeneous leisure preference parameter $k$, which are plotted in Figure 7. The estimated distributions indicates that leisure preferences are widely distributed among the population. Recall that a low $k$ implies a late retirement and vice versa. Appendix Figure 19 show, for four different types of individuals, how their expected retirement age varies across the k-distribution, and the plots clearly indicate that the different values of $k$ have a large impact on explaining the retirement ages predicted by the model.
8 Model Fit

8.1 In-sample Fit

The primary objective of our model is to predict the retirement behavior of seniors so that it can be used for policy analysis and forecasts. This, of course, requires that the model predictions fit the data reasonably well. One of the main advantages of our model approach is that we, for each person, compute an individual-specific distribution of the k-parameter. For an in-sample policy experiment, we can use these individual-specific preferences. However, to calculate these individual-specific k-distributions, we need to observe the actual retirement ages. If we want to use our model on a new set of individuals for whom we do not observe a chosen retirement age (e.g., in a forecast), we have to rely on the education- and gender-specific population distributions of p(k) depicted in Figure 7. As such, we can test the fit of our model in two different ways, using either individual-specific k-distributions or the
population distribution.

For each person, we compute the average probability distribution over the possible retirement ages \( r \in \{58, 59, ..., 72\} \). If we know individual \( j \)'s actual retirement age \( r^j \), we can calculate the probability that individual \( j \) retires at age \( r \) as: \( \Theta, x^j, k' \)

\[
P(r|\Theta, x^j, r^j) = \int_{\mathcal{X}} P \left( r|k, \hat{\Theta}, x^j \right) P(k|r^j, \hat{\Theta}, x^j) \, dk
\]

\[
= \int_{\mathcal{X}} P \left( r|k, \hat{\Theta}, x^j \right) \frac{P \left( r^j|k, \hat{\Theta}, x^j \right) p(k)}{P(r^j|\hat{\Theta}, \theta)} \, dk
\]

where we compute the integral numerically. However, if we do not know the actual retirement ages of the individuals, we instead use the estimated population distribution of \( p(k) \) to compute the probability that individual \( j \) retires at age \( r \) as:

\[
P(r|\Theta, x^j) = \int_{\mathcal{X}} P \left( r|k, \hat{\Theta}, x^j \right) p(k) \, dk
\]

Let \( N_r \) denote the expected number of individuals who retire at age \( r \). We sum the probability distribution over retirement ages for all individuals to get the expected number of retirements across the retirement ages:

\[
N_r^{\text{individual}} = \sum_{j=1}^{n} P(r|\hat{\Theta}, x^j, r^j),
\]

\[
N_r^{\text{population}} = \sum_{j=1}^{n} P(r|\hat{\Theta}, x^j)
\]

The graphs in Figure 8 show, for each gender- and education group, the model prediction fit for the cohorts 1942-1944, where we observe retirements up until age 72.\(^5\) The graphs plot the actual and predicted retirement distributions, relying both on both individual-specific and the aggregate population distributions of the leisure preference parameter \( k \). We see that both predictions fit the data very well, with very little difference in estimated exit rates. The models slightly under-predict the exit rates at age 62, most pronounced for low-educated men, with a slight corresponding over-prediction of exit rates at ages 61 and 63. There is substantial heterogeneity in retirement age distributions across the different education and group where women and individuals with shorter education retire earlier. While the model fit is slightly better for women and groups with longer education, our model performs well across all groups, even for the predictions that rely only on the population distributions of \( k \).

\(^5\)For the younger cohorts, we do not observe all of the possible retirement ages, but these cohorts fit the data equally well.
The graphs display the actual and predicted exit rates into retirement for birth cohorts 1942-1944 for whom we observe retirement exit rates from age 58-71. The vertical solid black lines plot the actual exit rates. The dashed black lines plot our model’s predicted retirement distribution when we use only the population distribution of k. The solid gray lines show the predicted retirement distribution modeled using the estimated individual-specific k-distributions.

Our model also fits data well across the distribution of private pension wealth. In Appendix F, Figures 20 and 21 show the model fits similar to that in Figure, but for the subset of individuals with pension wealth in the first and fourth qualities of the pension wealth distribution within each estimation group. These figures confirm the findings in Figure 5, which is that individuals respond to the financial incentives within the ERP scheme: Individuals holding low levels of pension wealth are more likely to retire at age 62, while individuals
with more pension wealth to a larger extent postpone their retirement to age 62, so they can benefit from a smaller means-testing in their ERP benefits with respect to their pension wealth. Our model provides very reasonable fits for both the low- and high pension wealth groups, and as such we believe that our model captures the heterogeneity in retirement ages across the different levels of pension wealth. Appendix Figure 22 shows the model fit for the subgroup of individuals who are not members of the ERP program. The figure shows that the model prediction based on individual-level $k$ distributions provide very good fits of the actual retirement age distribution, whereas the predictions that use the population distribution of $k$ tend to slightly over-predict the exit rates at earlier retirement ages (61-64) and under-predict the exit rates at later retirement ages (66-71). This is driven by the fact that the subgroup of individuals who are not members of the ERP scheme, counting only 10% of the estimation sample, on average has lower levels of the leisure preference parameter $k$.

### 8.2 External Validation

We perform an out-of-sample validation exercise to test if our model can predict the retirement decision of persons not included in the model estimation, and to whom different retirement rules apply. We estimated the model on the cohorts born in 1942-1952, and all of these face an early retirement age of 60 and a statutory old age pension age of 65. Now we use our estimated model to predict the retirement decision of individuals born in the second half of 1954, facing an early retirement age of 61 and a statutory old age pension age of 66. We change the NRA dummy accordingly, and thereby we assume that the focal point/social norm effect follows the policy rule change.\(^6\) As our data end in 2016, we can observe the share of individuals who retire before or on the early retirement age, 61. As we can only observe the chosen retirement ages at age 61 for individuals within the Early Retirement program (a self-financed retirement requires an extra year of observation), individuals who are not members of the Early Retirement scheme are excluded from the validation sample. Figure 9 shows the observed and predicted exit rates in the validation sample. The model does a good job predicting the exit rates at age 61 of the validation sample. For the groups with long tertiary education, the model fit is less precise. However, these predictions are more uncertain as they are based on few observations - 192 men and 128 women with long tertiary educations.

\(^6\)The 1954 cohort was 51 years old when the reform was adopted
The graphs display the actual and predicted exit rates into retirement for individuals born in the second half of 1954, who participate in the Early Retirement program. The vertical solid black lines plot the actual retirement age distribution. The dashed black lines plot our model’s predicted retirement distribution when we use only the population distribution of k.
9 Experiments

In this section, we present one baseline experiment in which we abolish the early retirement program, followed by three experiments where additional policy changes are added on top of the abolition of the early retirement program. We do this to mimic the retirement scene for future cohorts, where the large majority of individuals will not be eligible for ERP benefits. As such, the results of the main experiments will also reflect the prediction made by the baseline experiment. We run experiments within an experiment to get results which are more relevant for future policy making. All experiments are counterfactual by nature, as we conduct them on the estimation sample which includes the cohorts 1942-1952.

We will compare the estimated policy responses for individuals who hold different levels of retirement wealth. While earnings and non-retirement wealth are also important factors, these will not be the main focus of this analysis. We find that our conclusions are very similar once we restrict the analysis to only include average earning individuals. We also briefly show that non-retirement/liquid savings share many similarities with retirement savings in how it affects the response to the policy reforms.

A unique feature of our model is that we, for all persons, estimate an individual specific distribution of the leisure preference parameter $k$, which is retrieved from the person’s observed retirement age and financial circumstances. As such, the model allows us to include these individual-specific $k$-distributions when we do our counter-factual experiments. However, we find that the results of our policy experiments are similar when we use the estimated population distribution of $k$, rather than the individual-specific distributions.

9.1 Baseline Experiment

First, we simulate a baseline experiment in which we abolish the early retirement program and thereby mimic the retirement setting of future cohorts. Whereas the ERP scheme DKr covers the majority of individuals, recent changes to the program which have made it significantly less attractive combined with an opt-out option have led to a drastic drop in individuals entitled to receive ERP benefits. In Appendix Figure 22, we showed that our model also fit the retirement ages of individuals who are not members of the ERP scheme - the fit was best when we used individual $k$-distributions compared to the population distribution, but this distinction becomes less important when we consider the entire sample, and not just the selective group of individuals not entitled to ERP benefits.

Figure 10 shows the predicted exit rates for each retirement age when the Early Retirement Pension program is abolished, together with the in-sample model fit of the actual retirement rules of cohorts 1942-1952, where approximately 85% of the sample were entitled to receive ERP benefits starting at age 60. We see that the abolition of the ERP plan leads to a stark
increase in retirement ages according to our model. The peak in the exit rate at the ERP eligibility age (60) disappears, and the spike in the exit rate at the normal retirement age (65) drastically increases. The shift in exit rate spikes from 60 and 62 to 65 is strongest for those with lower education levels as expected since these groups were more inclined to retire on early retirement benefits. We also observe that the exit spike at age 62 (caused by the two-year rule) becomes a more smooth hump-shape in exit rates before the NRA. These are the individuals who decide to self-finance their retirement prior to when they become eligible for retirement benefits at age 65. The model predicts that 35-60% of seniors will choose to self-finance an earlier retirement, with largest shares for those with less education. While this share might seem very large, recall that we did observe a large proportion (approximately 40%) of those not eligible for ERP benefits retire prior to the NRA, see Figure 5. On average, our model predicts that individuals in the different gender- and education groups postpone their retirement by 0.5-1.5 years once the ERP scheme is abolished. The effect decreases when individuals hold large amounts of liquid and retirement wealth in an almost linear fashion, and the estimated effect is only 0.25-0.5 years for individuals with retirement savings equivalent to 9 or more years of pre-retirement income. This decline is illustrated in Appendix Figure 23, which also shows that the average effect of the reform is much larger for those with no or little education, and slightly larger for women compared to men.
The solid lines depict the model’s predicted exit rate distributions for the entire estimation sample consisting of cohorts 1942-1952 (these are similar to the model predictions shown in Figure 8, but include more cohorts). The dashed black lines plot our model’s predicted retirement distribution in the baseline experiment, where the ERP scheme is abolished. The solid gray lines show the predicted retirement distribution modeled using the estimated individual-specific k-distributions.
9.2 Postponing the NRA

As described in Section 2, the Old Age Pension (OAP) eligibility age has been indexed to the life expectancy of future cohorts in Denmark. Most other countries, including the U.S. and the U.K., have also increased normal retirement ages (NRAs) to improve their fiscal sustainability. When the NRA is postponed, governments will have to pay retirement benefits to the retirees for fewer years, but more importantly, an increase of the NRA also induces individuals to work longer, and thereby the labor force (and tax base) is increased. Understanding how an increase of the NRA affects the labor supply decision of seniors is, therefore, of very high importance.

An increase of the NRA corresponds to a decrease in the present value of discounted future OAP benefit payments, and does not impose strong financial incentives for individuals to respond to this change. However, as we assume that the social norm/focal point dummies also shift from age 65 to 66, we expect that the increase of the NRA will have a large impact on retirement ages.

Figure 11 shows, for each gender- and education group born in 1942-1952, the expected change in exit rates when the NRA is raised from age 65 to 66 within the baseline experiment where there is no ERP scheme. The figure shows that an increase of the NRA leads to a stark drop in the exit rate at the old NRA, and a corresponding increase in the exit rates of the new NRA. If we compare the estimated changes to the exit rates at age 65 in the baseline experiment, we see that the NRA spike is almost entirely shifted from age 65 to 66. However, a small fraction of individuals who would have retired at the normal retirement age (65) before the increase of the NRA choose to retire even earlier after the increase. Postponing the retirement age has almost no effect for the exit rates prior to age 62, but exit rates at 63 and 64 increase slightly by a few percentage points. This increase is driven by those individuals who no longer prefer to postpone their retirement until they reach the normal retirement age to gain the social norm/focal point jump in utility, as they would now have to wait one year longer. These are predominantly individuals with high estimated $k$ values. Consequently, the spike at the new OAP age, 66, is slightly smaller compared to when the NRA was 65. As expected, the increase of the NRA does not affect the exit rates at ages 66 and above.

\footnote{For individuals who are credit constrained, an increase of the NRA would impose strong financial incentives, but as our model assumptions imply that individuals can always borrow money against future income, this effect is not going to be captured by our model.}
Figure 11: Retirement Effect of One Year Increase of the NRA

The dashed black lines plot the model’s predicted retirement distribution in the baseline experiment, where the ERP scheme is abolished for cohorts 1942-1945. The solid gray lines show the predicted retirement distribution when we use the estimated individual-specific k-distributions.

As such, an increase of the NRA age from 65 to 66 make more individuals retire at the new normal retirement age 66, which increases the average retirement age. In fact, close to everyone who retired at the old NRA at 65 also retires at the new NRA at 66. However, this effect is offset by the increase in the number of individuals who no longer wants to wait to retire at the normal retirement age, and instead decide to finance an earlier retirement, resulting in increased exit rates at ages 63 and 64. Overall, an increase of the NRA from 65 to 66 increases the expected retirement ages of the groups by 0.06-0.15 years, with largest effects for those with low education, and larger effects for men.\(^8\) This number should be interpreted in the context of the baseline experiment, which predicts that 35-60\% of seniors will chose to self-finance an earlier retirement once the ERP scheme is abolished.

As self-financing an earlier retirement age is costly, one would expect individuals with larger private pension wealth to be more likely to do so. Figure 12 shows how the increase

\(^8\) Whereas the predicted changes in exit rate distributions are slightly different in a corresponding model without social norm dummies at the NRA (changes are more “smooth” compared to Figure 11), the aggregate effect as measured by increase in expected retirement age is very similar, also across different levels of private retirement savings.
in the expected retirement age, following a one-year increase of the NRA, vary for different levels of retirement wealth. Here we see a decline in the effect of postponing the retirement age of more than 75% when we compare the low retirement wealth individuals to those with large retirement savings. The decline stagnates at retirement savings levels equivalent to approximately four years of annual pre-retirement earnings. The figure also suggests that, conditional on the level of retirement savings, the expected effect of the increase is more significant for those with higher levels of education and slightly larger for men than women.

Of course, seniors can also finance an earlier retirement age using their liquid/non-retirement savings. Appendix Figure 24 shows that the slopes equivalent to those depicted in Figure 12 are even steeper for individuals who hold zero or negative liquid wealth. For individuals with zero liquid and retirement wealth, our model predicts that an increase of the NRA from 65 to 66 will result in an increase in expected retirement age by 0.25 years for men and 0.2 for women. For individuals holding large non-retirement wealth, exceeding 1.5 million DKr, the slope becomes flat - this is expected, given that these individuals do not have to rely on retirement savings to finance their early retirement.

Figure 12: Effect of one year increase in NRA by accumulated pension wealth

For each gender- and education specific group, the figure shows how our model predicts that an increase of the NRA from age 65 to 66 would have affected the expected retirement age of cohorts 1942-1952, given that the early retirement scheme was abolished. The effect is measured by years, as the change in expected retirement rates.
9.3 Reduced OAP benefit rates

Another strategy that countries use to improve the fiscal challenges of increased life expectancy is to decrease old age pension benefit rates. Effectively, such a policy is equivalent to a negative wealth shock to individuals similar to the one year increase of the NRA. Figure 15 plots, for each gender- and education group born in 1942-1952, the expected exit rates in the baseline experiment (solid gray line) with no ERP scheme together with the exit rates under an extended experiment (dashed black line) in which OAP benefits are reduced by five percent. As opposed to a one-year increase of the NRA, this experiment does not involve a change in the social norms/focal point of the retirement decision. This explains why the effect of the two types of reform differs. When OAP benefits are reduced by 5 percent, our model suggests that fewer individuals will retire prior to the NRA, and more individuals will retire after. The predicted changes in exit rates are small, but are distributed across all retirement ages - as there is a consistent drop at the retirement ages 58-65 and a consistent increase at ages 66-72, the accumulated effect is quite significant. Overall, the reduction in OAP payments increases the expected retirement ages of the groups by 0.05-0.09 years, with largest effect for those with low education, and larger effects for women compared to men.\textsuperscript{9} Compared to an increase in the NRA as shown in the previous experiment, a 5 percent reduction does not lead to a stark drop in exit rates at age 65. This is because a reduction in OAP benefits does not affect the credit-constrained individuals to the same extent as an increase in the NRA.

\textsuperscript{9}Results are similar in a model with no NRA social norm dummy, with the only difference being a slightly smaller spike at age 65 and a larger estimated increase in exit rates at age 66.
Figure 13: Retirement Effect of a five percent reduction of OAP-benefit rates

The figures plot, for each gender- and education specific group, the differences in predicted exit rates between the baseline experiment where the ERP scheme is abolished and an experiment in which the OAP benefit rates are reduced by 5%. The solid gray lines plot the change when we use individual-specific $k$-distributions, and the dashed black lines show the difference when the predictions are based on the population distribution of $k$.

Similar to the previous experiments, the estimated increase in the expected retirement age declines with increased levels of pension wealth, as pictured in figure 14. This means that individuals with larger retirement savings respond less to the policy change compared to those with low levels of accumulated retirement wealth. However, the decline in the policy response for increasing levels of retirement savings is much less steep when OAP benefits are reduced compared to when the NRA is increased. When the benefit rates are decreased by 5 percent, the model predicts that individuals with low levels of retirement savings will increase their retirement age by roughly 0.07 years for men and 0.09 years for women. The effect declines slightly to 0.05 years for men and 0.06 years for women who hold private retirement savings equivalent to four or more years of income at age 58. Thus, the two reforms (increasing the NRA by one year and decreasing OAP benefits by 5%) have roughly similar effects on the average retirement age for high-wealth individuals.
For each gender- and education specific group, the figure shows how our model predicts that a reduction in OAP benefits by 5% would have affected the expected retirement age of cohorts 1942-1952, given that the early retirement scheme was abolished. The effect is measured by years, as the change in expected retirement rates.

9.4 Reduced OAP means testing

As described in Section 4.2 (and in particular Figure 5), historical data suggests that the opportunity to avoid means testing of private pension wealth induced ERP eligible persons with pension wealth to postpone their retirement. In this subsection, we investigate the effect of a third policy, also targeted increased retirement ages, which is similar to the two-year rule of the Early Retirement Program. The suggested policy reduce the means testing of OAP benefits with respect to private pension wealth if retirement is postponed until at least age 66. Figure 15 shows, for each gender- and education group, the expected exit rates in the baseline experiment (solid gray line) with no ERP scheme together with the exit rates under an extended experiment (dashed black line) in which means testing of the OAP supplement with respect to private pension payments is avoided for three years if retirement is postponed to age 66 years or later (which is one year following the NRA).\textsuperscript{10} This experiment targets those who hold larger amounts of pension wealth, which mainly applies to workers with short-medium tertiary and long tertiary education. For these groups, the spike in retirement

\textsuperscript{10}Private pension payments above $\approx 70,000$ DKr are deducted from the pension supplement with a rate of 31 percent for singles and 16 percent for couples
at age 65 is reduced a little and moved to 66 years. Overall, the average expected retirement age is increased by 0.05-0.10 years. For unskilled and skilled workers, the effect is very small, because they have little pension wealth.\textsuperscript{11} The average expected retirement age increases by roughly 0.01 years.\textsuperscript{12}

Figure 15: Retirement Effect of reduced OAP means testing of private pension payments

The figures plot, for each gender- and education specific group, the differences in predicted exit rates between the baseline experiment where the ERP scheme is abolished and an experiment in which the ERP scheme is abolished and the means testing in OAP benefits by private pension payments is reduced when retirement is postponed to after the NRA. The solid gray lines plot the change when we use individual-specific $k$-distributions, and the dashed black lines show the difference when the predictions are based on the population distribution of $k$.

We would expect that individuals with large private pension wealth are more likely to respond to the experiment and postpone their retirement until age 66 or later, in order to avoid a means testing in their OAP benefits. As expected, Figure 16 shows an increasing trend in the predicted policy response, meaning that individuals holding large retirement savings respond stronger to the reform, compared those with less accumulated retirement

\textsuperscript{11}Mandatory pension savings plans was introduced for most groups of unskilled and skilled workers in the 1990s with contribution rates below 5 percent until 1998.

\textsuperscript{12}Results are similar in a model with no OAP social norm dummy.
savings. The predicted policy response is close to 0% for those holding low or no wealth, and increases to 0.05-0.09 for those holding retirement savings equivalent to six or more years of retirement savings. The effects of pension wealth are larger for workers with medium and tertiary educations.

Figure 16: Retirement Effect of Means Testing Relief

For each gender- and education specific group, the figure shows how our model predicts that a new means testing rule of OAP benefits with respect to private retirement income would have affected the expected retirement age of cohorts 1942-1952, given that the early retirement scheme was abolished (Means testing of the OAP supplement with respect to private pension payments is avoided for three years if retirement is postponed to age 66 years or later). The effect is measured by years, as the change in expected retirement rates.

10 A simpler model without leisure preference heterogeneity

The main contribution of our proposed model is that we allow for different individuals to have different preferences for leisure. If we instead assumed that the leisure preference parameter $k$ was a homogeneous parameter, our model would be very similar to a simplified dynamic programming model, where agents have perfect foresight in all respects except when they die. To find out whether the estimation of heterogeneous leisure preference parameters was worth the trouble, we estimate and test a similar model with a homogeneous $k$, and show how the model fit and experiment results differ compared to when we allow for leisure
preference heterogeneity. The results are shown in Appendix J. Table 4 shows the estimated parameters in the less flexible model, where the estimated point estimates of $k$ range from 0.9 to 1.3. Compared to the model with leisure preference heterogeneity, agents discount future consumption higher ($\beta$ coefficients are between 0.8 and 0.86), are more risk averse ($\rho$ between 1.6 and 2) and face lower attrition rates ($\alpha_0$ between 0.005-0.009). As expected, the less flexibility of the homogeneous $k$ specification of the model causes the log likelihood value to decrease from an average of -1.67 per person to -1.80 per person.

Figure 25 shows the model fit of the simpler model. As expected, the simpler model does not fit the data as well, and it predicts an almost flat and steady decrease in exit rates following age 60, however with a spike in exit rates at the NRA 65, caused by the social norm dummy. As such, the model does not seem to capture the spikes in exit rates at age 62, caused by the two-year of the ERP scheme. When we run the external validation test on the simple model, where we use the estimated model to predict the retirement decision of individuals born in the second half of 1954, facing an early retirement age of 61 and a statutory old age pension age of 66 (compared to 60 and 65 in the estimation sample), the model under-predicts the exit rate at age 61 by 30-50% for the different gender- and education groups. Consequently, we should not have much faith in the predictions of this simpler model.

For illustrative purposes, we also conduct the baseline experiment in which the ERP scheme is abolished for the simpler model without leisure preference heterogeneity, and show how the simpler model predicts that a one year increase of the NRA affects retirement ages. Figure 27 shows the actual predicted exit rates together with the predicted exit rates in the baseline experiment where the ERP scheme is abolished. The exit rates predicted by the simple model in the baseline experiment are almost flat across the ages 58-71, with a small curvature. Due to the social norm dummy, also this model predicts a large spike in the exit rates at the NRA, however smaller compared to those predicted by the model with leisure preference heterogeneity. The overall predicted increase in expected retirement ages following an abolition of the ERP scheme is slightly larger in the simpler model (0.5-2 years) compared to the model with leisure preference heterogeneity (0.5-1.5 years).

We also compare the predicted response to an increase of the NRA by the two models. The predicted changes in exit rates after an increase of the NRA from age 65 to 66 are shown in Figure 28. Similarly to the model with leisure preference heterogeneity, the simpler model also predicts that the exit rate shifts from the old to the new retirement age. The difference between the model predictions is that the simpler model does not predict that anyone decides to retire earlier, at ages 63 and 64, once the NRA is increased. This is mainly due to the fact that the simpler model have smaller estimated attrition parameters, $\alpha$, compared to the model with leisure preference parameters. As such, the simpler model suggest that the cost of remaining in the labor force for one more year is less costly, and as a result, the majority
of those who retired at the old NRA will shift to the new NRA after the increase. The model with preference heterogeneity estimates that attrition is much more important, with $\alpha$ parameters which are close to twice as large compared to those estimated by the simple model. The model with leisure preference heterogeneity is, therefore, suggesting that it is costly for individuals to retire late, which is offset by a large share of individuals having low values of $k$, making it costly to retire early as well. When individuals have different leisure preferences, they also respond differently to the policy changes. Whereas individuals with low leisure preferences are inclined to retire at the new NRA, individuals with large preferences for leisure are more inclined to choose to retire even earlier than before the increase in NRA, at ages 64 and 63.

Even though the simpler model predicts that a smaller share of individuals will respond to an increase of the NRA, the expected increase in average retirement age is still almost twice as large, as the model does not predict that some individuals will respond to the reform by decreasing their retirement age. Whereas the model with leisure preference heterogeneity predicted an overall increase in retirement ages of 0.06-0.15 years, with largest effects for those with low education, and larger effects for men, the simpler model predicts corresponding increases of the order 0.16-0.35 years.

Figure 29 shows how the simple model’s predicted increase in expected retirement ages, following a one-year increase of the NRA, vary for different levels of retirement wealth. We see a drastic decline in the effect of postponing the retirement age of roughly 50% from 0.3 years to 0.15 years when we compare the low wealth individuals to those with larger retirement wealth. The decline stagnates at savings levels of around four years of annual income prior to retirement. As such, the decrease of roughly 50% for large vs. small retirement savings predicted by the simple model is lower compared to the decrease predicted by the model with preference heterogeneity, where the effect decreased by roughly 75%.

11 Conclusion

We propose a novel structural retirement model of senior worker’s retirement decision, with heterogeneous leisure preferences, attrition, and improved health across generations. We estimate the model using high-quality Danish register data from 1996-2016, where we consider the retirement decisions of birth cohorts 1942-1954. We apply a non-parametric estimation technique to measure the heterogeneity in leisure preferences, and our estimates suggest that leisure preferences are widely distributed among the population. We find that the model fits the data well with reasonable parameter values. More importantly, we also find that our model provides decent predictions of an observed retirement response to an increase in the statutory retirement age in an external validation setting. Compared to a similar model without heterogeneous leisure preferences, we find that our proposed model performs
significantly better - both in terms of in-sample fit and external validation of the model.

We use our model to study the extent to which the increase in private pension wealth - providing individuals with more flexibility to choose their retirement age - impacts the effect of retirement reforms designed to increase the labor supply of seniors. We conduct several counter-factual experiments on the cohorts 1942-1952 which explore what their retirement decision would have been if different rules applied. First, we simulate a baseline experiment in which we abolish the ERP scheme. As the ERP scheme is being phased out, the baseline experiment mimics the retirement decision of future generations. We then contrast the retirement decisions in our baseline experiment to three additional experiments.

The first experiment increases the normal retirement age by one year from age 65 to 66. We find that individuals with zero private retirement savings delay their retirement with roughly 0.2 years for men and 0.15 years for women. For individuals with private retirement wealth equivalent to four or more years of pre-retirement earnings, the effect is reduced by roughly 75%. The second experiment reduces the old age pension benefits by five percent, and for this experiment, we estimate a decline in the expected retirement age of roughly 0.08-0.09 years for those with no retirement savings and 0.05-0.06 years for those with large retirement savings. As such, the effect of a decrease in retirement benefits is much more stable across different levels of retirement wealth as compared to an increase in the NRA. This is because a reduction in benefits is less consequential to credit-constrained individuals compared to a rise in the retirement age. We also consider a third policy experiment which introduces a means testing relief in the OAP which is similar to the two-year rule in the ERP-scheme: individuals can avoid a means testing of their old age pension with respect to their private pension income for three years if they retire one or more years after the statutory NRA. While low-wealth individuals are almost unaffected by this experiment, the expected retirement age increases by 0.06-0.10 years for individuals with large retirement savings. As such, this experiment - similar to the two-year rule of the retirement age - reverses the negative effect of pension wealth on retirement ages.

To summarize, our experiments find that the size of individual retirement savings can have important implications for the effect of different retirement reforms. For an increase in the NRA, we find a particularly large and negative effect of private retirement wealth on the reform’s ability to increase labor supply. It is possible, however, to reverse this effect, e.g. in a reform which mimics the two-year rule of the ERP scheme.
References

Søren Arnberg and Mikkel Barslund. The crowding-out effect of mandatory labour market pension schemes on private savings: Evidence from Renters in Denmark. 2014.


A Appendices

A Observed exit rates for ERP- and non ERP eligible, split on retirement wealth and earnings

Figure 17: Retirement by Pension Wealth and Earnings Quartile, ERP-eligible

The four figures plot the observed exit rates (share who exit the labor force) at ages 58-72 for individuals born in 1942-1944, who are included in our main sample (see Table 1), and who are eligible to retire on Early Retirement Pension benefits. The samples are split in pre-retirement earnings quartiles, and again in pension wealth quartiles, where quartile 2 and 3 are grouped together.
The four figures plot the observed exit rates (share who exit the labor force) at ages 58-72 for individuals born in 1942-1944, who are included in our main sample (see Table 1), and who are not eligible to retire on Early Retirement Pension benefits. The samples are split in pre-retirement earnings quartiles, and again in pension wealth quartiles, where quartile 2 and 3 are grouped together.
B Closed-form solution for optimal consumption

We would like to derive the indirect utility function of the consumer for a given retirement age \( r \). The consumer is faced with the problem defined by (1), (2) and (3) repeated below:

\[
U(r) = V(r) + \epsilon_r
\]

\[
V(r) = \sum_{a=58}^{120} \frac{(\gamma_a(r)c_a)^{1-\rho}}{1-\rho} D_a
\]  

\[
W_a = (1 + (1 - \tau)i_a)W_{a-1} + y_a(r) - c_a
\]

Given a retirement age \( r \), \( \epsilon_r \) is known by the consumer. According to (13) the consumer can then maximize \( U(r) \) just by maximizing \( V(r) \).

We will prove that maximizing \( V(r) \) given by (14) under the budget restriction, (15) yields the indirect utility function:

\[
V(r) = \left( \frac{W_{57} + H(r)}{P(r)} \right)^{1-\rho}
\]

where \( H(r) \) and \( P(r) \) are given by

\[
H(r) = \sum_{a=58}^{120} y_a(r) R_a
\]

\[
P(r) = \left[ \sum_{a=58}^{120} \gamma_a(r) D_a R_a^{\rho} \right]^{\frac{1}{\rho - 1}}
\]

Here \( R_a \) is defined as

\[
R_a = \prod_{s=58}^{a} \frac{1}{1 + (1 - \tau)i_a}
\]

**Proof:** We will start by demonstrating that we can transform the utility function (14) to a CES function. Define \( E = 1/\rho \) and the transformation \( T(V) = (1-\rho V)^{1-\rho} \). We have that \( T'(V) = \frac{1}{\phi} \left( \frac{1-\rho V}{1-\rho} \right)^{1-\rho} \) and that \( T'(V) > 0 \) iff. \( (1-\rho)V > 0 \). Observe that from (14):

\[
\hat{V}(r) \equiv T(V(r)) = \left[ \sum_{a=58}^{120} \gamma_a(r)c_a D_a \right]^{\frac{E-1}{E}}
\]

This is a CES utility function with elasticity of substitution \( E \). This transformation is only OK for \( T'(V) > 0 \). But this is always the case: assume \( 1 - \rho < 0 \). Then form (14) \( V < 0 \), such that \( (1 - \rho)V > 0 \). Assume \( 1 - \rho > 0 \). Then from (14) \( V > 0 \), such that \( (1 - \rho)V > 0 \). Due to continuity, it will also be the case for \( 1 - \rho = 0 \).

The flow condition given in the budget restriction in (15) can be re-written as a stock condition:
\[
\sum_{a=1}^{A} c_a R_a = W_{57} + H(r)
\]  

(17)

where \( R_a \) and \( H(r) \) are defined as above. Maximizing 16 given 17 we get (as a standard result for CES-functions) that

\[
c_a = \left( \gamma_a(r) \frac{E-1}{E} D_a \right)^E \left( \frac{R_a}{P(r)} \right)^{-E} \frac{W_{57} + H(r)}{P(r)}
\]  

(18)

Where

\[
P(r) = \left[ \sum_{a=58}^{120} \left( \gamma_a(r) \frac{E-1}{E} D_a \right)^E R_a^{1-E} \right]^{1\over 1-E}
\]

\[
= \left[ \sum_{a=58}^{120} \gamma_a(r) \frac{1-E}{\rho} D_a^{1\over \rho} R_a^{\rho-1} \right]^{\rho-1\over \rho-1}
\]

Inserting (18) in (16) we arrive at the indirect utility function

\[
\hat{V}(r) = \frac{W_{57} + H(r)}{P(r)}
\]

We then have that

\[
V(r) = T^{-1}(\hat{V}(r)) = \left( \frac{W_{57} + H(r)}{P(r)} \right)^{1-\rho} = \frac{(W_{57} + H(r))^{1-\rho}}{1 - \rho}.
\]
C Means-testing of the Early Retirement Pension (ERP)

Table 3: Annual Means Testing of ERP based on individual pension savings

<table>
<thead>
<tr>
<th></th>
<th>Retirement Before 62</th>
<th>Retirement After 62</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Annuities</td>
<td>NP</td>
<td>60% of (80% of RP - BA)</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>50% of AP</td>
</tr>
<tr>
<td>Term Pension</td>
<td>NP</td>
<td>60% of (5% of RD - BA)</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>50% of AP</td>
</tr>
<tr>
<td>Capital Pension</td>
<td>NP</td>
<td>60% of (5% of RD - BA)</td>
</tr>
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</table>

Note: NP = No Payments Made, P = With Payments, AP = Actual Payment (annual), RP = Reported Payment (annual), RD = Reported Deposited amount of total savings, BA = Basic Allowance (11.500 DKr in 2004) - can only be used once.
D Censored Observations

Recall our model assumptions about the retirement decision: the retirement age is decided upon at the beginning of age $a = 58$ where the agent has perfect foresight wrt. all future income. As long as we observe individual $j$ at age 57, we’re able to to compute $x_j = (W_{57}^j, H(58)^j, ..., H(72)^j)$ (see Section 5). Given the financial information available at age 57, we can simulate all future income streams for ages $a \in \{58, ..., 120\}$ given retirement age $r^j \in \{58, ..., 71\}$. As such, we can compute the choice set of all individuals despite the fact that we do not observe their chosen retirement age. Some individuals die or emigrate before retiring. For others, the data ends before a retirement age is observed - this could be the case for the youngest cohorts included in the analysis. However, we can still use the information that the individuals did not retire during the years where we observe them.

The probability of not retiring prior to the last observed age $a_d$ is given by:

$$
Prob_d \left( a_d | \Theta, x^j, k' \right) = \left( 1 - \sum_{r=58}^{a_d} Prob \left( r | \Theta, x^j, k' \right) \right)
$$

Consequently, the probability of an observation depends on whether or not the retirement age is observed, $a_d \leq r$. As such, the probability that individual $j$ either retire at age $r^j$ or do not retire prior to age $a_d^j$ is given by

$$
Prob \left( r^j | \Theta, x^j, k', a_d^j \right) = \begin{cases} 
\frac{exp(\nabla(r^j | \Theta, x^j, k'))}{\sum_{r'=58}^{72} exp(\nabla(r' | \Theta, x^j, k'))} & \text{if } a_d^j > r^j \\
\left(1 - \sum_{a=58}^{a_d^j} \frac{exp(\nabla(r | \Theta, x^j, k'))}{\sum_{r'=58}^{72} exp(\nabla(r' | \Theta, x^j, k'))}\right) & \text{if } a_d^j \leq r^j
\end{cases}
$$
The effect of the leisure preference parameter $k$ on expected retirement age

Figure 19: Expected retirement age as function of $k$ and estimated $k$-distributions

The figures show how different values of $k$ affect the expected retirement of four different types of individuals for each of the estimated gender- and education groups. We use the estimated homogeneous parameter estimates as presented in Table 2, and compute the expected retirement age predicted by the model for different values of $k$. The figures also show the estimated $k$-distribution histograms. Here, low income is set to 250,000 DKr at age 58, and a low pension wealth is set to 0 DKr. High income equals 500,000 DKr and a high pension wealth equals 500,000 DKr.
Figure 20: Predicted vs. Actual Exit Rates - Low Pension Wealth

The graphs display the actual and predicted exit rates into retirement for birth cohorts 1942-1944 with private pension wealth in the 1st quartile of the private pension wealth distribution within the group. The vertical black lines plot the actual retirement age distribution. The dashed black lines plot our model’s predicted retirement distribution when we use only the population distribution of k. The solid gray lines show the predicted retirement distribution modeled using the estimated individual-specific k-distributions.
Figure 21: Predicted vs. Actual Exit Rates - High Pension Wealth

The graphs display the actual and predicted exit rates into retirement for birth cohorts 1942-1944 with private pension wealth in the 4th quartile of the private pension wealth distribution within the group. The vertical black lines plot the actual retirement age distribution. The dashed black lines plot our model’s predicted retirement distribution when we use only the population distribution of k. The solid gray lines show the predicted retirement distribution modeled using the estimated individual-specific k-distributions.
G Model fit for individuals not entitled to early retirement benefits

Figure 22: Predicted vs. Actual Exit Rates - Not ERP eligible

The graphs display the actual and predicted exit rates into retirement for birth cohorts 1942-1944 who are not members of the Early Retirement Program. The vertical black lines plot the actual retirement age distribution. The dashed black lines plot our model's predicted retirement distribution when we use only the population distribution of k. The solid gray lines show the predicted retirement distribution modeled using the estimated individual-specific k-distributions.
Effect of baseline experiment across different levels of private retirement wealth

Figure 23: Effect of one year increase in NRA by accumulated pension wealth

For each gender- and education specific group, the figure shows how our model predicts that abolishing the ERP scheme would have affected the expected retirement age of cohorts 1942-1952, given that the early retirement scheme was abolished. The effect is measured by years, as the change in expected retirement rates.
I How non-retirement wealth affects the predicted response to an increase of the NRA

Figure 24: Effect of one year increase in NRA by accumulated pension wealth

The figure plots the predicted effect of an increase of the NRA from age 65 to 66 on the expected retirement age of cohorts 1942-1952, assuming that the early retirement scheme was abolished. Split by gender, the black dots depict the estimated effect for everyone by level of pension wealth, the black triangles depict the effect for everyone holding zero or negative wealth, and the black squares show the effect for the subsample holding more than 1.5 mio. DKr in non-retirement/liquid wealth.
### Table 4: Estimated Homogeneous Model Parameters (without leisure preference heterogeneity)

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<th>N</th>
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Figure 25: Predicted vs. Actual Exit Rates (without leisure preference heterogeneity)

The graphs display the actual and predicted exit rates into retirement for birth cohorts 1942-1944, using the simpler model without leisure preference heterogeneity. The vertical black lines plot the actual retirement age distribution. The dashed black lines plot our model’s predicted retirement distribution when we use only the population distribution of k.
Figure 26: Predicted vs. actual exit rates, external validation test on cohort 1954 (without leisure preference heterogeneity)

The graphs display the actual and predicted exit rates into retirement for individuals born in the second half of 1954, who participate in the Early Retirement program, using the simple model without leisure preference heterogeneity. The vertical solid black lines plot the actual retirement age distribution. The dashed black lines plot our model’s predicted retirement distribution when we use only the population distribution of k.
The solid lines depict the model’s predicted exit rate distributions for the entire estimation sample consisting of cohorts 1942-1952 using the simple model without leisure preference heterogeneity (these are similar to those plotted in 25, but include more cohorts). The dashed black lines plot the simpler model’s predicted retirement distribution in the baseline experiment, where the ERP scheme is abolished.
The figures plot, for each gender- and education specific group, the differences in predicted exit rates between the baseline experiment where the ERP scheme is abolished and an experiment in which the ERP scheme is abolished and the NRA is increased from 65 to 66, when we use the simple model without leisure preference heterogeneity to predict.
Figure 29: Retirement Effect of One Year Increase of the NRA (without leisure preference heterogeneity)

For each gender- and education specific group, the figure shows how the simple model without leisure preference heterogeneity predicts that an increase of the NRA from age 65 to 66 would have affected the expected retirement age of cohorts 1942-1952, given that the early retirement scheme was abolished. The effect is measured by years, as the change in expected retirement rates.